

# dxFeed FX Risk Factor Parity Index Methodology

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## 1 Summary

The *dxFeed FX Risk Factor Parity Index*<sup>TM</sup> (“PARIS”, *Parity of Risk*) reflects the value of a currency basket equally exposed to three risk factors: “*Carry*”, “*Value*” and “*Momentum*”.

The choice of these risk factors is influenced by the current discussion in academic literature; see section 5 for details. Carry is associated with the asset’s yield, Value with its fundamental price, while Momentum summarizes its recent price movements. Analysis shows that the three factors combined explain a large proportion of the FX rate dynamics (average  $R^2$  across currencies up to 60%). The index weights are optimized so that the resulting currency basket is equally robust to volatility shocks induced by the three risk factors; see section 6 for details.

The index is targeted at investment professionals interested in tracking the performance of the “safe haven” currency basket with a preference towards controllable risk allocation across different investment styles.

## 2 Index Model

The index value is the effective growth rate of a currency basket, in percentage terms.

In particular, let  $C = \{\text{USD, EUR, GBP, } \dots\}$  be a set of currencies comprising the index. For a currency  $c \in C$ , denote as follows:

$\text{Spot}_c(t)$  is the spot rate of the currency  $c$  against USD at time  $t$ . This is a direct quote in American terms—it increases in value when the currency appreciates. For USD itself,  $\text{Spot}_{\text{USD}}(t)$  is the value of the Dollar Index.

$\text{Weight}_c$  is the weight of currency  $c$ .

Then the corresponding 1-period return is  $\text{Return}_c(t) = \text{Spot}_c(t)/\text{Spot}_c(t-1) - 1$  and the basket return is

$$\text{Return}(t) = \sum_{c \in C} \text{Return}_c(t) \cdot \text{Weight}_c.$$

The effective growth rate is calculated recursively as

$$\begin{aligned} \text{Growth}(t) &= (1 + \text{Return}(t_0))(1 + \text{Return}(t_0 + 1)) \dots (1 + \text{Return}(t - 1))(1 + \text{Return}(t)). \\ &= \text{Growth}(t - 1) \cdot (1 + \text{Return}(t)). \end{aligned}$$

However for calculation convenience, a closed-form approximation for log-returns is used to compute the actual index value at time  $t$ :

$$\widehat{\text{Growth}}(t) = \prod_{c \in C} \text{Spot}_c^{\text{Weight}_c}(t).$$

Finally, the index value is

$$\text{Index}(t) = \frac{100}{\text{Divisor}} \widehat{\text{Growth}}(t).$$

Divisor is an index parameter chosen to achieve index value continuity between rebalancings. It is initially equal to  $\widehat{\text{Growth}}(t_0)$ , where  $t_0$  is the time when the index was first computed.

## 3 Component Selection

The set of index components

$$C = \{\text{USD, EUR, GBP, } \dots\}$$

consists of 20 currencies obtained as follows. The initial list of 39 major currencies is acquired from The Bank for International Settlements<sup>1</sup>. The following currencies are then removed from consideration:

<sup>1</sup><https://www.bis.org/triennial.htm?m=2677>

- Pegged currencies: AED, SAR, HKD, DKK, BGN, BHD.
- Low turnover currencies: COP, PHP, RON, PEN, ARS.
- Currencies with extraordinary volatility and/or very unusual past events: MYR, THB, CNY, TRY, RUB.
- Currencies, for which no data was obtained for the purpose of risk factor construction: SGD, TWD, IDR.

## 4 Parameter Derivation

### 4.1 Weights

The set of index weights  $\{\text{Weight}_c, c \in C\}$  is derived so that the resulting currency basket is equally exposed to the three risk factors: Carry, Value, and Momentum. See section 6 for details. Naturally,

$$\sum_{c \in C} \text{Weight}_c = 1, \text{Weight}_c \in [0, 1] \quad \forall c \in C.$$

### 4.2 Divisor

At each rebalancing at time  $t_r$ , the new divisor is computed as

$$\text{Divisor} = \frac{100}{\text{Index}'(t_r)} \widehat{\text{Growth}}(t_r),$$

where  $\text{Index}'(t_r)$  is computed using the old parameters, and  $\widehat{\text{Growth}}(t_r)$  using new parameters.

## 5 FX Risk Factors

We generally follow [Baku et al., 2019] for the construction of FX risk factors. Their idea is to explain currency returns under the linear model with market risk factors directly extracted from FX markets—a “Carhart model” for currencies. The rationale behind using market risk factors (as opposed to the traditional economic approach based on monetary policy, inflation, etc) is that under the assumption of efficient markets, market data contains all the available information, including the economic risk factors; moreover, the data is available at frequent intervals, which is important for parameter estimation procedures.

The three factors—Carry, Value and Momentum—are constructed identically:

1. A measure is identified which is used to score every eligible currency proportionally to the expected return.
2. A currency basket is composed, consisting of  $N$  long and  $N$  short positions according to the score values.
3. The currency basket return is computed using spot prices as a weighted average of the two legs, i.e.

$$\frac{n_\ell}{n_\ell + n_s} \sum_{c \in C_\ell} w_c R_c(t) + \frac{n_s}{n_\ell + n_s} \sum_{c \in C_s} w_c R_c(t),$$

where  $C_\ell, C_s$  are the sets of long, short leg components,  $n_\ell = |C_\ell|$ ,  $n_s = |C_s|$ ,  $w_c$  is the weight and  $R_c(t)$  is the return of component  $c$ .

One major difference in comparison with [Baku et al., 2019] is that when a degree of freedom is present it is considered a hyperparameter. The value of the hyperparameter is chosen to maximize the mean  $R^2$  of the 3-factor model. The factor hyperparameters are specified in the corresponding sections below. For portfolio

construction, we optimize for the leg size  $N$  and the weighting scheme: equiweighted or in proportion to the measured value.

Another difference is in how the factor portfolio is constructed: only the currencies with positive (resp. negative) measure values are considered as the long (resp. short) portfolio leg constituents. Because of that, the number of leg components isn't always equal to  $N$ .

## 5.1 Carry

[Kojien et al., 2018] suggests that the return in each of the major asset classes<sup>2</sup> can be decomposed as

$$\text{Return} = \text{Carry} + E(\text{Price appreciation}) + \text{Unexpected price shock.}$$

“Carry” is defined as futures return if prices stay the same. Unlike the “Price appreciation” term which is estimated using a specific asset price model, Carry is directly observable from futures prices. According to the analysis in [Kojien et al., 2018], Carry is a strong positive predictor of returns, generating positive and unexplained alpha relative to other known factors.

At time  $t$ , denote as follows:

Spot( $t$ ) is the spot price;

Forward( $t$ ) is the price of a futures contract expiring at  $t + 1$ ;

Capital( $t$ ) is the amount of capital allocated to finance each futures contract;

IR( $t$ ) is the domestic risk-free rate.

Then at  $t + 1$  the position's price is

$$\text{Capital}(t)(1 + \text{IR}(t)) + (\text{Forward}(t + 1) - \text{Forward}(t))$$

and the total return is

$$\begin{aligned} \text{ReturnTotal}(t) &= \frac{\text{Capital}(t)(1 + \text{IR}(t)) + (\text{Forward}(t + 1) - \text{Forward}(t)) - \text{Capital}(t)}{\text{Capital}(t)} \\ &= \frac{\text{Forward}(t + 1) - \text{Forward}(t)}{\text{Capital}(t)} + \text{IR}(t). \end{aligned}$$

Under the assumption that the spot price will stay the same (Spot( $t + 1$ ) = Spot( $t$ )), and since the futures expires at the future spot price (Forward( $t + 1$ ) = Spot( $t + 1$ )), the return in excess of the risk-free rate is the *carry*

$$\text{Carry}(t) = \frac{\text{Spot}(t) - \text{Forward}(t)}{\text{Capital}(t)},$$

which scales depending on the amount of capital Capital( $t$ ). If the position is fully collateralised, Capital( $t$ ) = Forward( $t$ ) and

$$\boxed{\text{Carry}(t) = \frac{\text{Spot}(t)}{\text{Forward}(t)} - 1.} \quad (1)$$

To reiterate, this definition of carry is a term in the equation that explains an asset's price dynamics, and hence can be used to compute the factor portfolio value. However, it does match the classical definition of FX carry: let IR\*( $t$ ) be the foreign interest rate; then by *Covered Interest Rate Parity* (CIRP),

$$\text{Forward}(t) = \text{Spot}(t) \frac{1 + \text{IR}(t)}{1 + \text{IR}^*(t)}$$

<sup>2</sup>Currencies, equities, bonds, commodities, treasuries, credit, call/put index options.

and, substituting into 1,

$$\text{Carry}(t) = \frac{\text{IR}^*(t) - \text{IR}(t)}{1 + \text{IR}(t)}.$$

In other words, Carry is approximately equal to the interest rate differential. If  $\text{IR}^*(t) > \text{IR}(t)$ , then it is profitable to convert local currency into foreign, earn interest and convert back (assuming CIRP holds)—positive carry.

We directly apply 1 to compute the Carry measure. At any time  $t$ , several futures contracts are available for a given currency with different expiration dates. For index construction this is regarded as hyperparameter  $T_{\text{fut}}$  which is optimized to maximize mean  $R^2$  in-sample.

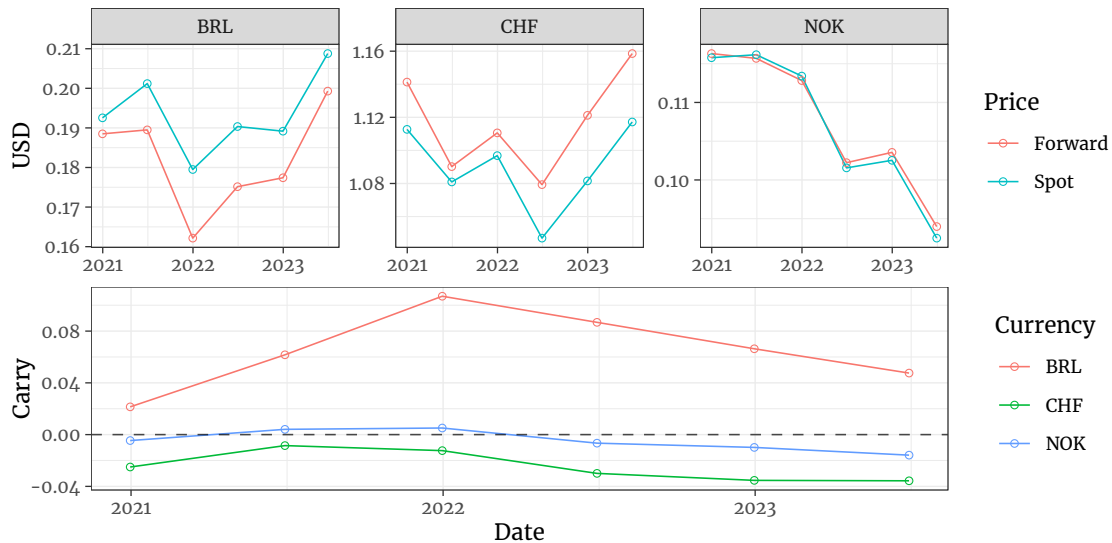


Figure 1: An illustration of how Carry is computed for three currencies (BRL, CHF, NOK) over 3 years every 6 months;  $T_{\text{fut}} = 12$  months

## 5.2 Value

Let  $\text{Spot}(t)$  be the spot and  $\text{Fair}(t)$  be the hypothetical “fair” price of an asset. The *Value* measure is defined as

$$\text{Value}(t) = \frac{\text{Fair}(t)}{\text{Spot}(t)} - 1.$$

If  $\text{Fair}(t) > \text{Spot}(t)$ , the asset is underpriced and  $\text{Value}(t) > 0$ ; its price is expected to grow, hence a long position is expected to generate positive return.

[Baku et al., 2019] review and calculate a range of economic models to measure the “fair value” of a currency. We adopt the one based on the principle of *Purchasing Power Parity* (PPP). Other models may give better results, especially if aggregated. However some of them (e.g. BEER, FEER) are highly correlated with PPP, therefore for practical purposes implementing just one is sufficient.

Let

$\text{Spot}_{c^*/c}(t)$  be the direct spot exchange rate of foreign currency  $c^*$  against domestic currency  $c$  at time  $t$ .

$\text{Price}_c(t)$  be the price level of the country corresponding to a currency  $c$ , e.g. CPI, a wholesale price index, a GDP deflator, etc.

Then the bilateral Real Exchange Rate is the spot rate adjusted for relative price levels of the two countries:

$$\text{RER}_{c^*/c}(t) = \frac{\text{Price}_{c^*}(t)}{\text{Price}_c(t)} \text{Spot}_{c^*/c}(t).$$

$\text{RER}_{c^*/c}$  increases when the domestic currency  $c$  depreciates in real terms.<sup>3</sup>

The Real Effective Exchange Rate is the multilateral RER, computed as the geometric average of bilateral RER of  $c$  against relevant currencies/countries  $\mathcal{B}$ :

$$\text{REER}_c(t) = \prod_{c^* \in \mathcal{B}} \text{RER}_{c^*/c}^{w_{c^*}}(t).$$

The set of relevant currencies/countries  $\mathcal{B}$  and the weights  $w_{c^*}(t)$  is computed differently depending on the methodology. For example, the Bank for International Settlements (BIS)<sup>4</sup> derives the weights from manufacturing trade flows among trade partners, and the weights vary on a three-year basis.

For the purpose of index construction we use REER values computed and disseminated by BIS. Contrary to the definition above, BIS computes  $\text{REER}_c^{-1}(t)$ , so an increase in REER corresponds to an appreciation of  $c$  in real terms, rather than a depreciation.

According to the notion of the Purchasing Power Parity, currencies are in equilibrium if similar goods have the same prices in two countries, and the exchange rates adjust to reflect the difference between inflation rates among countries. Similarly, REER must periodically return to its long-term average  $\overline{\text{REER}}_c(t)$ , which is a proxy for the equilibrium REER. Therefore, the Value measure becomes

$$\text{Value}_c(t) = \overline{\text{REER}}_c(t) / \text{REER}_c(t) - 1.$$

The amount of data to calculate  $\overline{\text{REER}}_c(t)$  is a hyperparameter  $T_{\text{REER}}$  which is optimized to achieve the maximum mean  $R^2$  in-sample.

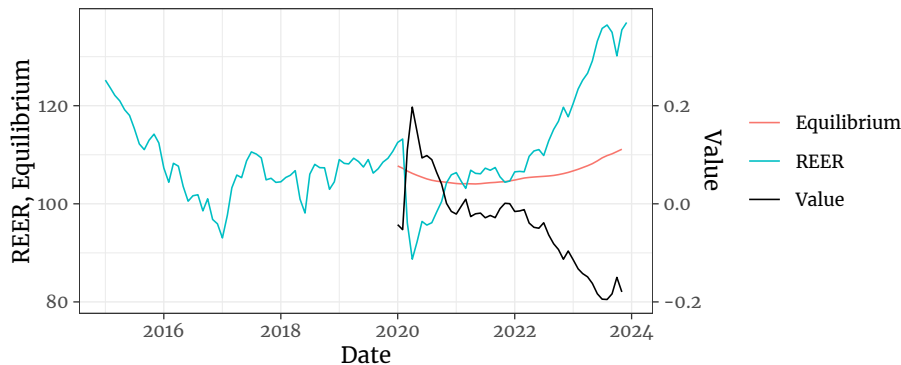


Figure 2: An illustration of REER, Equilibrium REER & the Value measure computed for MXN using monthly data from BIS. The MA period  $T_{\text{REER}}$  was set to 5 years.

<sup>3</sup>For example, let  $c^*$  be EUR and  $c$  be USD and suppose  $\text{Price}_c(t)$  is the price of a burger. Let  $\text{Price}_{\text{USD}}(t_0) = 1$ ,  $\text{Price}_{\text{EUR}}(t_0) = 1$  and  $\text{Spot}_{\text{EUR}/\text{USD}}(t_0) = 1$ .

- If  $\text{Spot}_{\text{EUR}/\text{USD}}(t_1) = 1.2$ , then  $\text{RER}_{\text{EUR}/\text{USD}}(t_1) = 1.2$ —an increase in RER indicates a depreciation of the domestic currency (USD) in real terms, because 1 EUR buys 20% more burger.
- If  $\text{Price}_{\text{USD}}(t_2) = 0.8$ , then  $\text{RER}_{\text{EUR}/\text{USD}}(t_2) = 1 \cdot 1.2/0.8 = 1.5$ —now it's even cheaper to buy a burger in the US, and 1 EUR buys 50% more of it.

<sup>4</sup>See <https://data.bis.org/topics/EER>.

### 5.3 Momentum

The Momentum is a well-known risk factor in several asset classes; see [Baku et al., 2019, p. 29] for a brief overview.

Let as before  $\text{Spot}_c(t)$  be the spot direct exchange rate of  $c \in C$  against USD. Then the  $c$ 's momentum at time  $t$  with horizon  $h$  is simply its past return

$$\text{Return}_c(t, h) = \frac{\text{Spot}_c(t)}{\text{Spot}_c(t - h)} - 1.$$

To obtain the Momentum measure, [Baku et al., 2019] suggest averaging the momenta over a predefined set of horizons  $\{h_1, \dots, h_M\}$ :

$$\text{Momentum}_c(t) = \frac{1}{M} \sum_{i=1}^M \text{Return}_c(t, h_i).$$

In particular, they suggest computing Momenta with horizons equal to 1, 3 and 12 months (short-, medium- and long-term Momenta).

The above design is used with two modifications:

- To reduce the effect of market noise,  $\text{Spot}_c(t)$  is defined on calendar months and yields the mean of daily close rates for this month.
- The set of horizons  $\{h_1, \dots, h_M\}$  was chosen to maximize the mean  $R^2$  of the resulting factor.

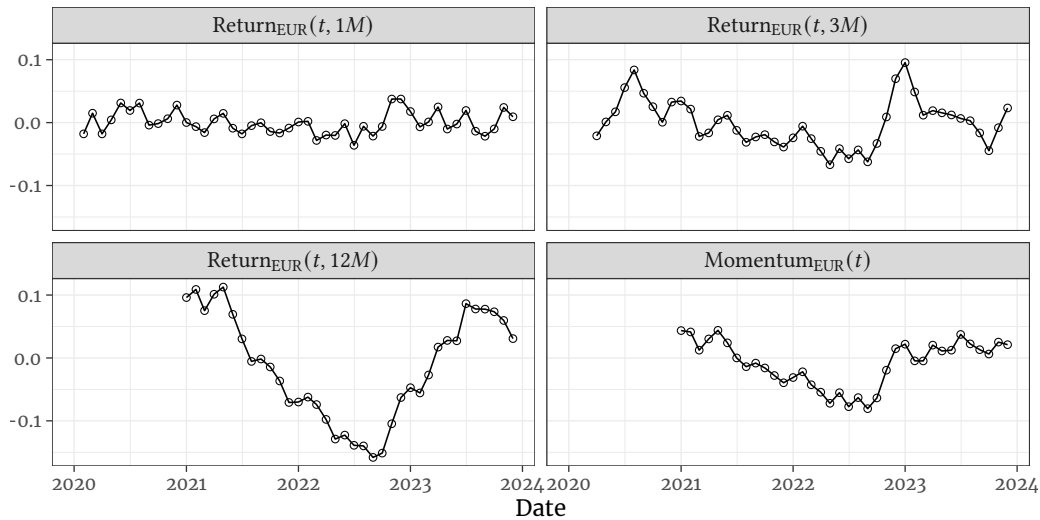


Figure 3: An illustration of averaging varying-horizon EUR/USD returns to obtain the Momentum measure.

In line with how other factor portfolios are constructed, only currencies with positive (negative) Momentum measure are considered as long (short) leg constituents. This corresponds to the “time-series” Momentum of [Baku et al., 2019]. Empirical analysis shows, that the “cross-section” Momentum portfolio (that does consider components regardless of the measure sign) introduces a high correlation with other factors, in particular with the “time-series” Momentum. Although it helps raise the total  $R^2$  somewhat, the high correlation often causes the risk factor parity optimisation to fail, therefore we omit this version of the factor from subsequent consideration.

The particular values of  $\{h_1, \dots, h_M\}$  are regarded as hyperparameters and are optimized to maximize mean  $R^2$  in-sample.

## 5.4 The 3-factor Model

*Remark.* The figures below are presented for illustration purposes and may not match the actual figures used for index construction. See the most recent factsheet for the up-to-date information.

The three-factor portfolios are rebalanced with fixed periodicity. Fig. 4 shows the composition of each factor.



Figure 4: Factor composition per rebalancing.

The dynamics of currency returns are modeled using the following linear regression, which is standard for this type of factor models:

$$\text{Return}_c(t) = \alpha_c + \sum_{f \in \mathcal{F}} \beta_{f,c} \text{Factor}_f(t) + \epsilon_c, \quad \mathbb{E}\epsilon_c = 0, \text{cov}(\epsilon_c, \epsilon_{c'}) = 0, \text{cov}(\text{Factor}_f, \epsilon_c) = 0.$$

The sensitivity coefficients  $\beta_{f,c}$  apply to the entire study period (“static estimation”). For many currencies the coefficients were statistically significant (different from 0), indicating that the corresponding currencies act as important predictors for currency returns (Fig. 5).

The coefficients of determination  $R^2$  range from near-zero values to as high as 80% for some currencies at certain time periods. The model does a poor job explaining currency return dynamics when the market regime changes significantly due to major macroeconomic/geopolitical events, e.g. the COVID-19 outbreak or the beginning of the conflict in Ukraine at the end of February 2022; see figure 6.

## 6 Risk Factor Parity Optimisation

[Roncalli, 2013] introduces a portfolio optimisation method based on risk factor parity. It may be regarded as a way to diversify a portfolio against the risks induced by a linear factor model. Given a factor model that explains the return dynamics of a selected group of assets, it is possible to “budget” the risks associated with every factor, so that the resulting portfolio is exposed to the given risk factors desirably.

The method described below works for a broad range of asset classes. For index construction it is applied to currency factors given in section 5.



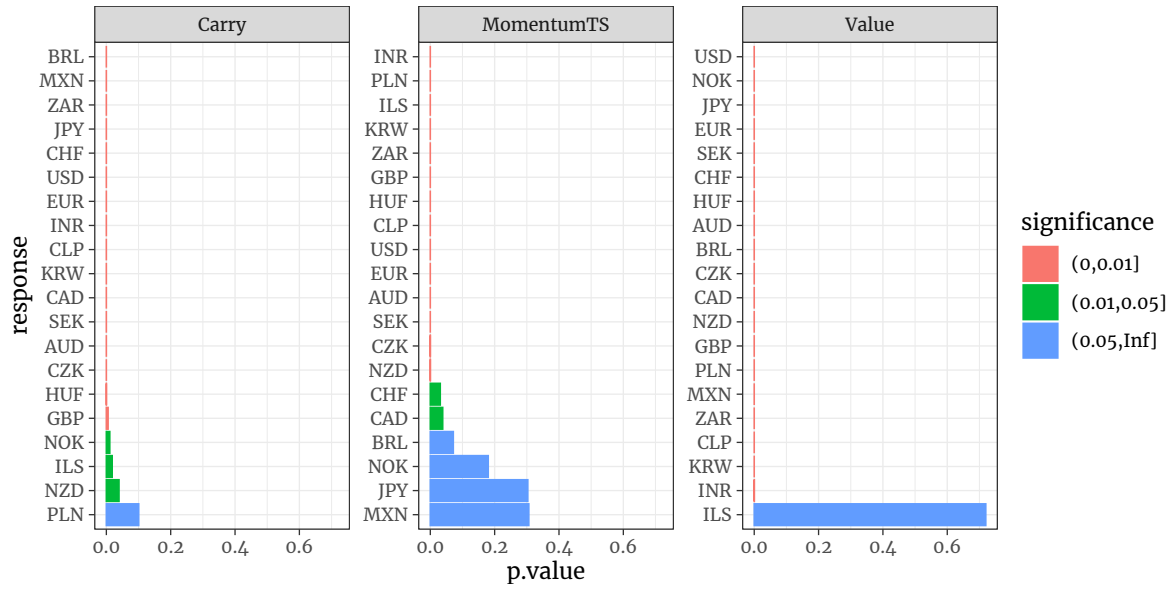


Figure 5: Significance of factor loadings  $\beta_{f,c}$  per factor per currency. Low p-values indicate high significance of the corresponding predictor. The intercept term  $\alpha_c$  isn't shown (insignificant for all the currencies).

## 6.1 Linear Factor Model

Assume the following linear model:

$$R_t = \mathbf{A}\mathcal{F}_t + \epsilon_t, \quad t \in [t_r - T_{FM}, t_r] \quad (2)$$

where

$R_t \in \mathbb{R}^n$  is the (random) vector of daily returns for the  $n$  currencies at time step  $t$ ,

$\mathcal{F}_t \in \mathbb{R}^m$  is the (random) vector of the  $m$  risk factor daily returns at time step  $t$ ,

$\mathbf{A}_{n \times m}$  is the loadings matrix,

$\epsilon_t \in \mathbb{R}^n$  is the residual vector, uncorrelated with  $\mathcal{F}_t$ ,  $E\epsilon_t = \mathbf{0}$ .

At rebalancing date  $t_r$ , estimate  $\mathbf{A}$  using  $T_{FM}$  days of training data for  $R_t, \mathcal{F}_t$ .

For stability reasons, instead of using sample estimate of covariance matrix  $\Sigma$ , use an estimate induced by the model 2, namely

$$\text{cov } R_t = \Sigma = \mathbf{A}\mathbf{Q}\mathbf{A}^\top + \mathbf{D}, \quad \mathbf{D} = \text{cov } \epsilon_t = \text{diag}(\sigma_1^2, \dots, \sigma_n^2), \quad \mathbf{Q} = \text{cov } \mathcal{F}_t.$$

## 6.2 Risk Decomposition

Enumerate the index components and let  $\mathbf{w}$  be the weight vector corresponding to the weights  $\text{Weight}_c$ ,  $c \in \mathcal{C}$ :

$$\mathbf{w} \in \mathbb{W} = \{\mathbf{w} \in \mathbb{R}^n \mid \mathbf{1}^\top \mathbf{w} = 1, \mathbf{w} \geq \mathbf{0}\}.$$

Let  $\mathcal{R}(\mathbf{w})$  denote a portfolio risk measure, e.g. portfolio volatility  $\mathcal{R}(\mathbf{w}) = \sqrt{\mathbf{w}^\top \Sigma \mathbf{w}}$ <sup>5</sup>. The marginal risk contribution  $\partial \mathcal{R}(\mathbf{w}) / \partial w_j$  shows the sensitivity of the portfolio risk to increasing the  $j$ -th component weight,  $j \in 1 : n$ .

<sup>5</sup>Other examples of risk measures include Value-at-Risk and Expected Shortfall. However, under the normal model, the relative risk contribution is the same for all the measures. Moreover, both the VaR and ES require an estimation of the expected portfolio return, which is often unreliable. Therefore we choose to work with the volatility risk measure.

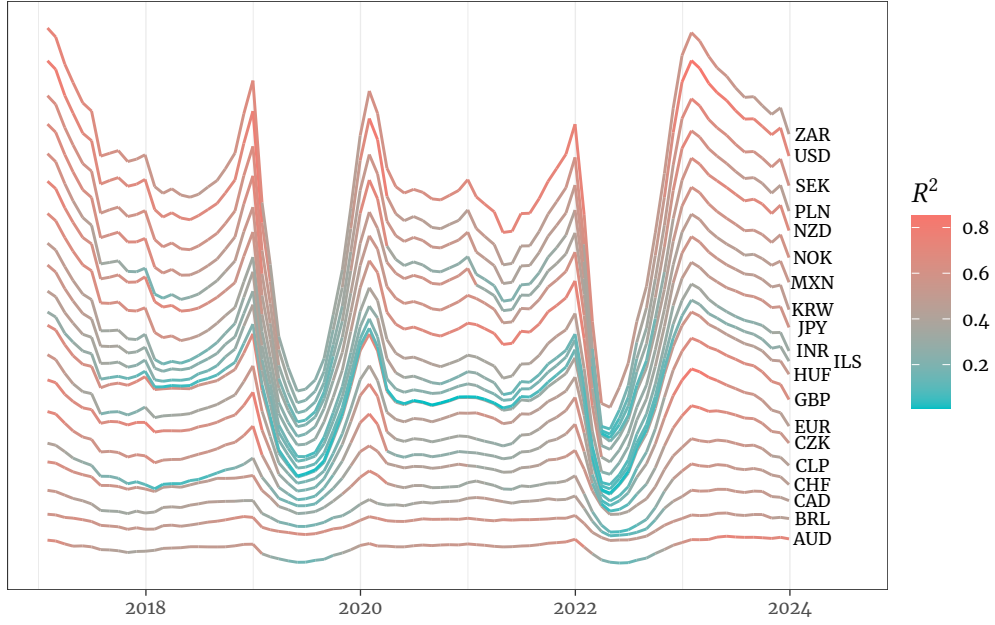


Figure 6: Rolling 1-year  $R^2$  of the factor model per currency.

Denote asset weights as  $\mathbf{w}$  and factor weights (exposures) as  $\mathbf{u}$ . The asset weight can be decomposed as

$$\mathbf{w} = \mathbf{B}^+ \mathbf{u} + \tilde{\mathbf{B}}^+ \tilde{\mathbf{u}},$$

where  $\mathbf{B}^+ = (\mathbf{A}^\top)^+$  (Moore–Penrose inverse)  $\tilde{\mathbf{B}} = \ker(\mathbf{A}^\top)^\top$ ,  $\mathbf{u} = \mathbf{A}^\top \mathbf{w}$ ,  $\tilde{\mathbf{u}} = \tilde{\mathbf{B}} \mathbf{w}$ . Importantly,  $\tilde{\mathbf{B}}^+ \tilde{\mathbf{u}}$  is the residual and  $\tilde{\mathbf{u}}$ , being a vector of residual factors, has no economic interpretation. The gradient vector of marginal risk can then be decomposed as

$$\frac{\partial \mathcal{R}(\mathbf{w})}{\partial \mathbf{w}} = \left( \frac{\partial \mathcal{R}(\mathbf{w})}{\partial \mathbf{u}} \mathbf{B}, \frac{\partial \mathcal{R}(\mathbf{w})}{\partial \tilde{\mathbf{u}}} \tilde{\mathbf{B}} \right)^\top,$$

where

$$\frac{\partial \mathcal{R}(\mathbf{w})}{\partial u_i} = \left( \mathbf{A}^+ \frac{\partial \mathcal{R}(\mathbf{w})}{\partial \mathbf{w}} \right)_i, \quad \frac{\partial \mathcal{R}(\mathbf{w})}{\partial \tilde{u}_i} = \left( \tilde{\mathbf{B}} \frac{\partial \mathcal{R}(\mathbf{w})}{\partial \mathbf{w}} \right)_i.$$

Let  $\text{RFC}_i(\mathbf{w})$  denote the risk contribution of the  $i$ -the factor to the total portfolio risk. The total risk can then be decomposed into a sum of risk factor contributions

$$\mathcal{R}(\mathbf{w}) = \sum_{i=1}^m \text{RFC}_i(\mathbf{w}) + \sum_{i=m+1}^n \widetilde{\text{RFC}}_i(\mathbf{w}),$$

where

$$\text{RFC}_i(\mathbf{w}) = u_i \frac{\partial \mathcal{R}(\mathbf{w})}{\partial u_i} = (\mathbf{A}^\top \mathbf{w})_i \cdot \left( \mathbf{A}^+ \frac{\partial \mathcal{R}(\mathbf{w})}{\partial \mathbf{w}} \right)_i,$$

$$\widetilde{\text{RFC}}_i(\mathbf{w}) = \tilde{u}_i \frac{\partial \mathcal{R}(\mathbf{w})}{\partial \tilde{u}_i} = (\tilde{\mathbf{B}} \mathbf{w})_i \cdot \left( \tilde{\mathbf{B}} \frac{\partial \mathcal{R}(\mathbf{w})}{\partial \mathbf{w}} \right)_i.$$

### 6.3 Risk Factor Parity Portfolio

Let  $b_i$  be the admissible “budget” of the  $i$ -the risk factor,  $\sum_{i=1}^m b_i = 1$ , e.g.  $b_i = 1/m$  means all factors have equal budget. The goal is to find portfolio weights so that the  $i$ -the risk factor contribution has the

pre-defined share of the total risk:

$$\text{RFC}_i(\mathbf{w}) = b_i \mathcal{R}(\mathbf{w}), \quad i \in 1 : m. \quad (3)$$

For that, [Roncalli, 2013] suggests to minimize the sum of squared differences over the risk factors (ignoring the residual risk factor contributions), i.e.

$$\begin{pmatrix} \mathbf{u}^* \\ \tilde{\mathbf{u}}^* \end{pmatrix} = \arg \min_{\mathbf{w} \in \mathbb{W}} f(\mathbf{w} | \mathbf{b}), \quad f(\mathbf{w} | \mathbf{b}) = \sum_{i=1}^m (\text{RFC}_i(\mathbf{w}) - b_i \mathcal{R}(\mathbf{w}))^2. \quad (4)$$

Numerical minimisation of 4 is performed until a solution sufficiently close to 0 is obtained (in which case the budgeting equation 3 holds), or a maximum number of iterations is reached (which usually yields a sub-optimal solution, i.e. a portfolio satisfying the given conditions cannot be possibly found).

## 7 Lifecycle & Maintenance

### 7.1 Rebalancing

The indices' composition is reviewed periodically with periodicity  $T_R$ —see the accompanying factsheet for details. Rebalancing might occur as a result of such a review.

Rebalancing procedures yield a new set of components  $C$  and their corresponding parameters (Weights and Divisor); see Sections 3 and 4.

### 7.2 Symbol Removal

In the unlikely case that a currency  $c^*$  ceases to exist or is deemed no longer suitable for index construction, the default behavior is to use its last known value to compute the index (“last observation carried forward”). The index composition will be updated before or at the next scheduled rebalancing, at the discretion of the steering committee.

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# Revision History

Apr 1, 2024 Initial release.

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